SECONDARY MATHEMATICS TEACHERS' ASSOCIATION

(SMATA)

20 22

O'LEVEL MATHEMATICS SEMINAR

THE 6th ANNUAL GRAND MATHS ZONAL SEMINAR 2022
ORGANIZED AT
ST. JOSEPH OF NAZARETH HIGH SCHOOL

Saturday 6th June, 2022

PAPER ONE 456/1

- Statistics
- Matrices
- Transformations
- Linear Programming
- Quadratics
- Inequalities
- Construction
- Bearings
- Trigonometry
- Probability
- Operations
- Fractions
- Factorisation
- Equations & Formulae
- Circle Properties

NOTE

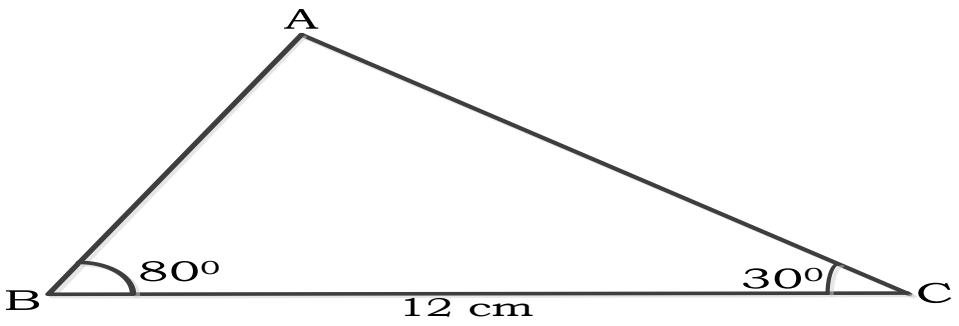
SMATA MATHS JOINT MOCK EXAMS 5th August, 2022

PAPER TWO 456/2

- B Set theory
- Vectors
- Business Math (Taxation)
- Kinematics
- Mensuration
- 3 Dimensions
- Mapping
- Functions
- Coordinate Geometry
- Ratios & Proportions
- Variations
- Decimals
- Surds
- Indices
- Logarithms

(A) GENERAL QUESTIONS

1. Given that $a * b = a - b^2a + ba$, evaluate;
   (i) $2 * 2$
   (ii) find the value of m for which $3 * m = 3$.
2. Without using mathematical tables or calculator, evaluate
   $27^{\frac{2}{3}} \times \left(\frac{81}{16}\right)^{\frac{-1}{4}}$
3. Form a quadratic equation where roots are $^{-1}/_{2}$ and $^{3}/_{4}$.
4. Simplify: $\frac{1}{2}\log_{10} 64 + \log_{10} 25 - \log_{10} 2$
5. Under an enlargement scale factor $1\frac{1}{2}$ and centre (0,0) the point A (-4,2) is mapped onto A^1. Determine the coordinates of A^1.
6. If $\underset{\sim}{a} = \begin{pmatrix}3\\2\end{pmatrix}$ $\underset{\sim}{b} = \begin{pmatrix}-1\\4\end{pmatrix}$ and $\underset{\sim}{c} = \begin{pmatrix}5\\3\end{pmatrix}$. Find the;
   (i) $\underset{\sim}{a} + 2\underset{\sim}{b} + \underset{\sim}{c}$ (ii) $\left|\frac{1}{2}\underset{\sim}{a} - \underset{\sim}{b}\right|$
7. In the triangle ABC below, BC = 12 cm, $\angle ABC = 80^0$ and $\angle ACB = 30^0$.



Calculate ,the area of the triangle ABC, correct to 4 significant figures.

8. Mr. Odeke bought a Smart phone at a cash discount of 10%. Given that the marked price was sh. 360,000/=.

(i). Find the actual amount he paid for it.

(ii). If he sold it to Amongi at a loss at a loss of 20%. Find how much did Amongi paid him.

9. Make P the subject of the expression, $M^2 = \frac{P^2 + R}{3 - 2P^2}$

10. A lorry is moving at 60 kmh^{-1} and a Prado car which is 50 km behind it is moving in the same direction at 80 kmh^{-1}.

Calculate the;

(i) Time taken by the Prado car to catch up with the lorry.

(ii) Distance travelled by the Prado car to catch up with the lorry.

(B) MATRICES

11. Solve the simultaneous below using matrix method

$\frac{x}{4} = \frac{y}{3} + \frac{4}{3}$ and $\frac{x}{3} + \frac{1}{6} + \frac{y}{2} = 0$

12. Given that matrix $P = \begin{pmatrix} 3x^2 & 8 \\ 3x & 4 \end{pmatrix}$. Find the value of x for which matrix P is singular.

13. If A $= \begin{pmatrix} 3 & 5 \\ 4 & 7 \end{pmatrix}$ and B $= \begin{pmatrix} 2 & 3 \\ -1 & 5 \end{pmatrix}$. Find;

(i) C if C $= A^{-1}B$

(ii) C + A + 2B

14. Shakirah is a business woman who deals in Agricultural produce in a certain week, she visited four markets.

In market A, she bought 5 bags of beans, 3 boys of maize, 8 bags of irish potatoes and 2 bags of millet. In market B, she bought 2 bags of beans, 3 bags of irish potatoes and 1 bag of millet.
In market C, she bought 4 bags of beans, 5 bags of irish potatoes and 2 bags of millet. In market D, she bought, 5 bags of beans, 3 bags of maize, 3 bags of irish potatoes and 3 bags of millet.
She bought each bag of beans at shs. 300,000, a bag of maize at shs. 150,000 a bag of irish potatoes at shs. 100,000 and a bag of millet at shs. 185,000.
She later sold all the produce she bought at shs. 350,000 per bag of beans, shs. 180,000 per bag of maize, shs. 145,000 per bag of irish potatoes and shs. 200,000 per bag of millet.

(a) (i) From a 4 x 4 matrix for the produce Shakirah bought from the four markets.

(ii). Form a 4 x 1 cost matrix for the price of the produce.

(iii) By matrix multiplication, find the amount of money spent on the produce in each market.

b) (i) Form a 4 x 1 sales matrix for the price at which produce was sold.

(ii). Find the amount of revenue got from the sale of the produce, hence find her profits.

(C) INDICES, SURDS AND LOGARITHMS

15. Solve for x in the equation $\left(\frac{1}{9}\right)^{x} \div 243 = \left(\frac{1}{27}\right)^{1-x}$
16. Solve for x : $\log_{10}(x+3) - \log_{10}(x-5) = 1$
17. By using mathematical tables, evaluate $\frac{(1934)^2 x \sqrt{0.00324}}{436}$ to three significant figures
18 Simplify : $\sqrt{128} + \sqrt{18} + \sqrt{98} - \sqrt{72}$
19. Express $\frac{6-\sqrt{2}}{\sqrt{2}+1}$ in the form $a + b\sqrt{2}$. Hence, find the value of a and b.

(D) QUADRATICS

20. (a) Draw a graph of the function $y + 3x^2 - 4x - 4 = 0$ for the interval $-3 \leq x \leq 4$.

Use your graph to state the roots of the equation.

(i) $-3x^2 + 4x + 4 = 0$

(ii) $6x - 6x^2 + 4 = 0$

(b) State the line of symmetry of the function.

(E) SET THEORY AND LOGIC

21. In a certain company; a sample of 100 employee's was picked randomly. In this sample; it was found out that 78 employee's eat Fish (F); 82 employees eat meat (M); 53 employee's eat beans (B); and 2 do not eat any of the three items. All those that eat Beans also eat meat. 48 employee's eat all the three items.

(a) Represent the given information on a Venn diagram.

(b) How many employee's eat;

(i) atleast two items

(ii) atmost two items

(iii) both Fish and meat but not Beans

(c) If an employee is picked at random; from the sample, What is the probability that the employee eats;

(i) Only two items.

(ii) Fish or Beans.

22. In market meeting, it was proposed that a total of 45 traders should sell the following; Banana's (B), Cassava (C) and Peas (p). It was found that; 23 sell ***B*** **,** 24 sell ***P*** and 20 sell ***C.*** n(BnPn C^1) = 7, n(BnC) = 8 and n(PnC) = 13. The number of trader's who sell none of these items is three times the number of those who sell C only.

(a) Present the information on a Venn diagram.

(b) Find the; (i) $n(BnC^1nP^1)$ (ii) $n(PnC^1nB^1)$

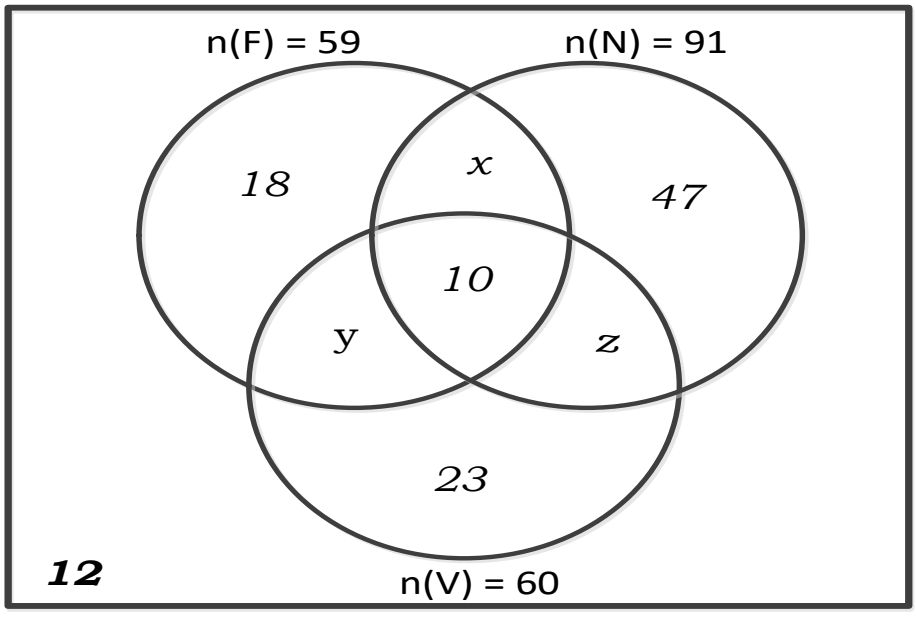
(iii) $n(CnB^1nP^1)$

(c) If a trader is picked at random, find the probability of he/she,

(i) selling all the three items

(ii) selling $n(P^1nC^1nB^1)$

23. The Venn diagram below shows participants in different games; Netball (N), Volley ball (V) and Football (F).



(a) Determine the values of x, y and z.

(b) Find the number of participants, who did;

(i) play atmost one game

(ii) play atleast two games

(iii) only two games

(c) Find the probability that a participant picked at random, plays;

(i) N or V

(ii) N and V

(iii) Both F and V only

(F) STATISTICS

24. The table below shows that marks obtained by students in senior four of a certain school in a Biology exam marked out of 100.

32	38	42	57	91	70	78
42	53	44	62	71	61	77
38	44	79	33	78	63	63
51	48	56	55	75	48	61
70	61	88	90	71	78	68
70	53	72	68	72	69	89
54	53	56	68	78	49	39
89	79	69	59	60	59	49

(a) (i) Draw a frequency distribution table starting with 30 as the lower-class limit and using equal intervals of 10.

(b) State (i) the number of students in the class

(ii). Median class

(iii) Modal class

(c) Calculate the; (i) mean mark.

(ii) median mark. (**correct to 2 dps**)

(d) Draw a histogram and use it to estimate the modal mark.

(e) Find the probability that a student picked at random scored 60% and above.

25. Okoth invests 400,000/= at the beginning of each year in a microfinance. The money accumulates at a compound rate of 14% per annum.
    (i) Find his total amount immediately after payment of her third investment
    (ii) Interest earned.

26. At the beginning of the 2020, Mr. Mugimba deposited Shs. 2,200,000 in a bank which offers a compound interest rate of 2.5% per four months. Find how much interest he earned at the end of the year.

27. A salesman receives a commission, which is a percentage of his sales as follows:-

Sales (Shs)	**Commission (%)**
0 - 200,000/=	2
Any amount above 200,000/=	15

Find the total commission for sales worth Shs. 800,000.

28. The above is an advert at a shop in Kampala tagged on several Computers.

COMPUTERS COMPUTERS COMPUTERS !!!

SOLD AT SHS. **1,000,000**.
PAY CASH AND SAVE **10%**
OR;
PAY IT EASILY BY HIRE PURCHASE
BY MAKING A DOWN PAYMENT OF **20%** OF THE VALUE OF GOODS AND THEN PAY 12 MONTHLY EQUAL INSTALLMENTS OF **8%** OF THE VALUE OF GOODS.

Mr.Kamugisha went to the shop wanting to buy a new computer. How much would he save by buying the computer on a cash basis instead of buying it on the hire purchase scheme?

29. Beads with letters are arranged such that they form the word **MISSISSIPPI**. Find the probability that a bead picked contains;

    (i) a letter S.

    (ii) a letter that is a consonant.

30. A dice numbered from 1 to 6 is tossed with a coin once.

    (a) write down the possible sample space.

    (b) find the probability of getting.

        (i) a head and a prime number.

        (ii) a tail and an even number.

31. The table below shows the multiplication of numbers.

X	2	3	4
5	–	15	–
6	12	–	–
8	–	24	–

Complete the table and hence, find the probability that the product is even.

32. A box contains 4 black, 3 green and 5 red pens. If two pens are picked at random without replacement. Find the probability that;

    (i) The two pens are red.

    (ii) Both pens are of the same colour.

    (iii) Atleast a black pen is picked.

    (iv) No black pen is picked.

    (v) How many green pens must be added to the box so that the probability of picking a green pen is 1/2 ?

33. (a) Given that $h(x) = x^2 - 4$ and $f(x) = -2x + 2$. Find
(i) $hf(x)$ (ii) $ff(-1)$

(b) If $g(y) = 5 + 2y$; $gh(y) = -4y + 7$ and that $h(y)$ is linear; find (i) $h(y)$ (ii) $hg(y)$

(iii) the value of y for which; 2hg (y) = 3gh(y)

34. (a) Given the functions; g(x) = $\frac{1-3x}{2}$, $f(x) = \frac{x+2}{2}$. Find,

(i) $g^{-1}(x)$ (ii) $g^{-1}f(x)$ (iii) g^{-1}f(0) .

(ii) the value of x for which fg(x) = $\frac{4x-5}{3}$,

(iii) If $h(x)$ $\frac{3x+5}{3x^2+5x-2}$; find the values of x for which h(x) is not defined.

35. (a) Given that; $g(y) = ay^2 + by + 1$; $g(1) = 6$ and $g(2) = 15$;
Find the values of a and b.

(b) Given the domain X = {2,3,5,7,8,9,10}.

(i) Draw a papygram of x showing; "is a factor of ".

(ii) Using an arrow diagram show it's corresponding range if x is mapped onto $-x^2 + 100$. State the type of mapping obtained.

(J) TRIGONOMETRY

36. If angle θ is reflex and tan θ = $\frac{-5}{12}$, find without using tables, the value of (i) $\sin\theta$ (ii) cos θ - 2sin θ.

37. Musisi is 2 metres tall and standing vertically ***d*** metres away from the foot of a flag pole. The angle of elevation of the top of the flag pole from Musisi's head is 40°. Musisi walks 8 metres towards

38. the foot of the flag pole and finds that the angle of elevation has increased by 30°. Calculate the height, **h** of the flag pole and the distance, **d** of the base of the flag pole from Musisi's original position?

(K) THREE DIMENSIONS (3D SHAPES)

39. The figure below is a rectangular-based pyramid VPQRS in which PQ = 30 cm and QR = 2Ocm. V is a point such that VP = VQ = VR = VS = 60 cm and PO = OR.

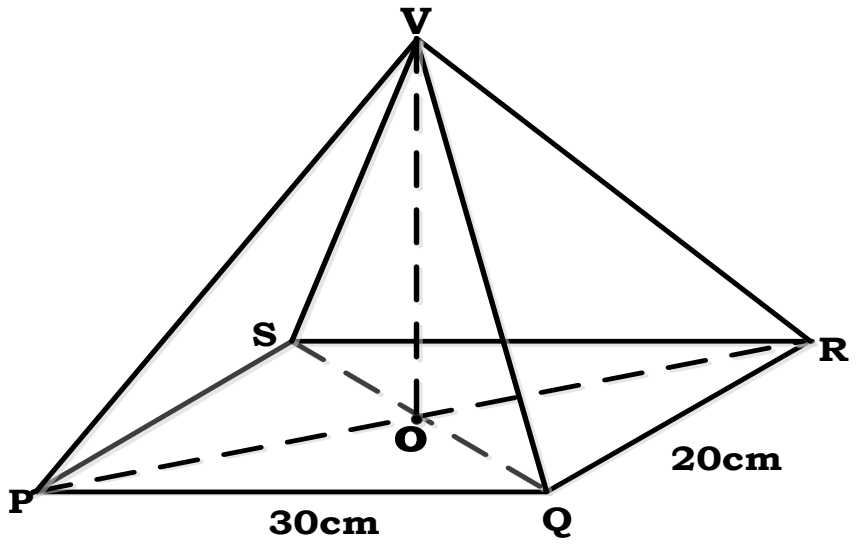


Fig..4

Find:

(a) What is the projection of line VR on the plane PQRS.

(b) (i) The vertical height VO of the pyramid.
(ii) The length of PR.

(c) The angle
(i) between line VP and plane PQRS.
(ii). between line VP and VR.
(iii) The angle between the planes VPS and VQR

(d) The volume and hence, the capacity of the pyramid.

(L) INEQUALITIES, REGIONS & LINEAR PROGRAMMING

40. Find all the integral values of x which satisfy the inequalities

$$2(2-x) < 4x - 9 < x + 11$$

41. The manager of a hotel has sufficient money to buy a total of 100 crates of sodas, Mirinda Apple and Mirinda fruity. He wants to buy at least twice as many crates of Mirinda –apple as Mirinda fruity. He wants to buy a maximum of 80 crates of Mirinda-apple and atleast 10 crates of Mirinda -fruity.

Taking x to be the number of crates of Mirinda Apple and y for Mirinda fruity, write down all the inequalities based on these facts. Show these inequalities on a graph and outline the region in which (x,y) must lie. The profit on a crate of Mirinda Apple and Mirinda fruity is shs 2300 and 2500 respectively. Find the number of crates of Each type that he should buy to make maximum profit and calculate this maximum profit.

(M) TAXATION

42. Below is a table showing the tax structure for a certain country. The Taxable income of all government employees is as follows;-

Income per month (shs)	**Rate (%)**
0 - 20,000	Tax free
20,001 - 60,000	10
60,001 – 140,000	15
140,000 – 300,000	20
300,001 and above	25

Mr. Wambede is a school bursar ;he pays income tax of Ug Shs.44,400 . He is entitled to the following allowances;

Transport 50,000 per month, medical 912,000 per annum, lunch 24,000 per month, water 40,000 per month, housing 1.8 million per annum.

An employee is given a family allowance for any three of his children according to age distribution.

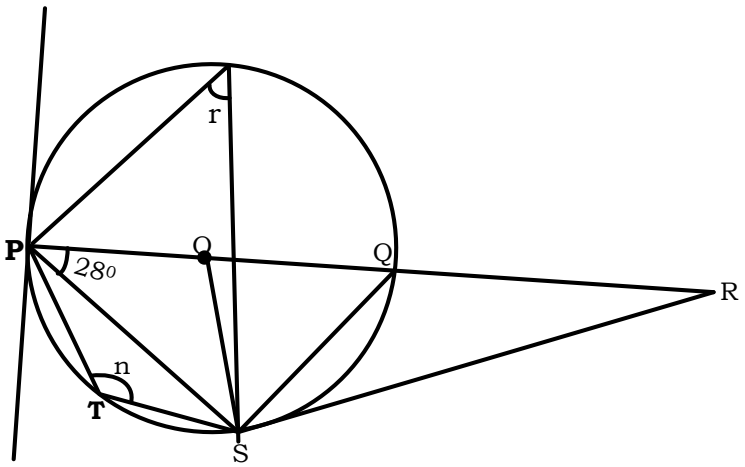
Age (years)	Ugshs
0-12	6000
13-18	4000
19-21	2000

Given that Mr. Wambede has a family of two children with a daughter aged 8 years and a son of 20 years old.

(a) Calculate his taxable income
(b) Calculate his gross income
(c) Determine the percentage of his gross income that goes to tax.
(d) If Mr. Wambede was to be paid salary in dollars, how much would it be to the nearest dollar?
[1 dollar = Ug. Shs. 3,700]

(N) CIRCLE PROPERTIES

43. In the figure below angle ∠ SPO = 28, If O, is the centre of the circle with a tangent at P and S.



(a) Determine then following angles.

(i) obtuse angle POS (ii) angle OQS

(iii). angle SRQ (iv) angle QSR

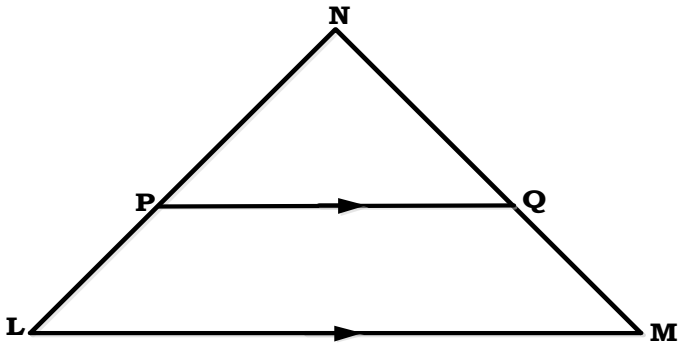
(v). angle PTS (vi) angle n and r

(b) Given that length SR = 16 cm and length PR = 19 cm. Find length QR.

(P) SIMILARITIES

44. Two similar container's have capacities of 1.571 litres and 12.568 litres. If the height of the smaller is 20cm; find the height of the bigger container.

45. The figure below shows a triangle LMN, PQ is Parallel to LM

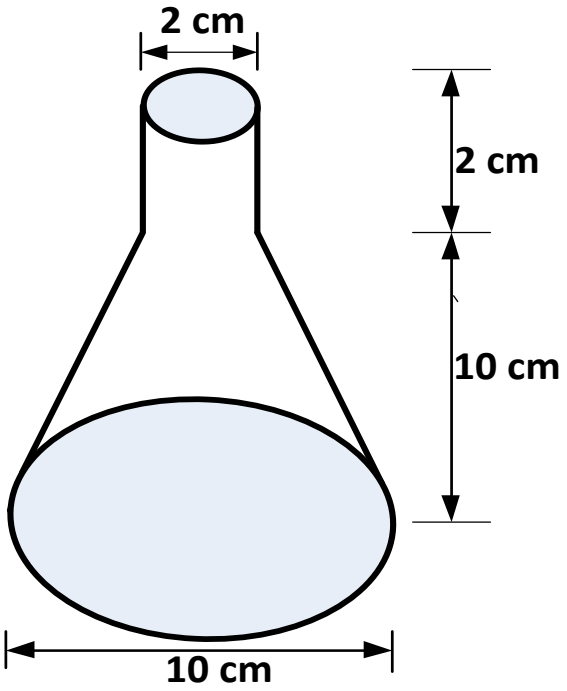


If PQ = 10cm; LM = 15cm and NQ = 6cm. Find the ratio for the area of triangle NPQ to area of quadrilateral LPQM. Given that the area of triangle NLM is 45 cm^3.

46. A cylindrical tank of diameter 1.4m and height 2m has a capacity of 3.08m^3. Find the radius and height of a similar tank of capacity 83.16m^3.

(Q) MENSURATION

47. The figure below represents a conical flask. The flask consists of a cylindrical part and a frustrum of a cone. The diameter of the base is 10 cm while that of the neck is 2 cm. The vertical height of the flask is 12 cm.



Calculate, correct to **1** decimal place;

(i) The slant height of the frustrum part

(ii) The slant height of the smaller cone that was cut off to make the frustrum part.

(iii) The external surface area of the flask (Take $\pi = 3.142$)

(iv) The amount of water in litres the flask can hold when full.

(R) BEARING

48. Three towns Arua, Busia and Soroti are such that Busia and Soroti are on a bearing of 070^{o} and 155^{o} from Arua respectively. Also, Soroti is on a bearing of 210^{o} from Busia. Arua and Busia are 520km a part.

(a) Using a scale of 1cm to represent 50km draw a diagram to scale showing the location of each of the three towns.

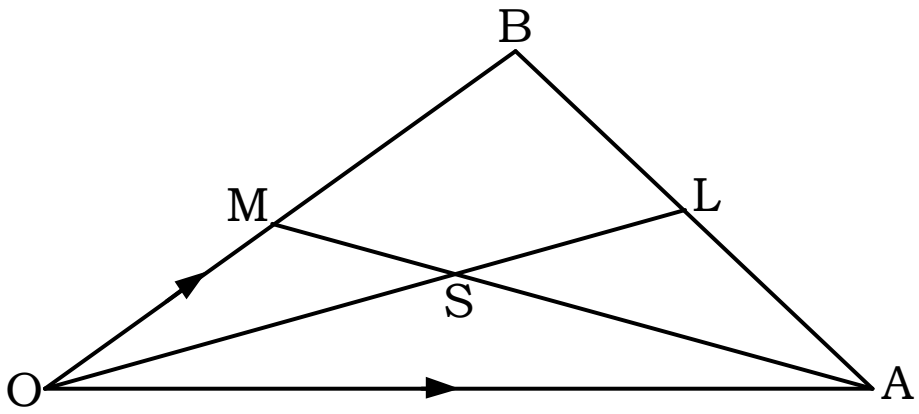
(b) Using your diagram above, find the distance of Soroti from Arua and Busia respectively.

(c) What is the angle of elevation of a pilot flying a plane 300km vertically above Arua from Soroti?

(S) VECTORS

49. OXY is a triangle with position vectors OX = $\underset{\sim}{x}$; OY = $\underset{\sim}{y}$. Express in terms of $\boldsymbol{x}$ and **y** the position vector of Z if Z divides XY in the ratio of 1:2

50. In triangle OAB below, $\overrightarrow{\boldsymbol{OA}} = \underset{\sim}{\boldsymbol{a}}$ $\overrightarrow{\boldsymbol{OB}} = \underset{\sim}{\boldsymbol{b}}$. A point "L" is on the side AB and "M" on the side OB. OL and AM meet at "S". $\overrightarrow{AS}$ = SM and $\overrightarrow{OS} = \frac{3}{4}\overrightarrow{OL}$. Given that; $\overrightarrow{OM} = x\,\overrightarrow{OB}$ and $\overrightarrow{AL}$ = y $\overrightarrow{AB}$.



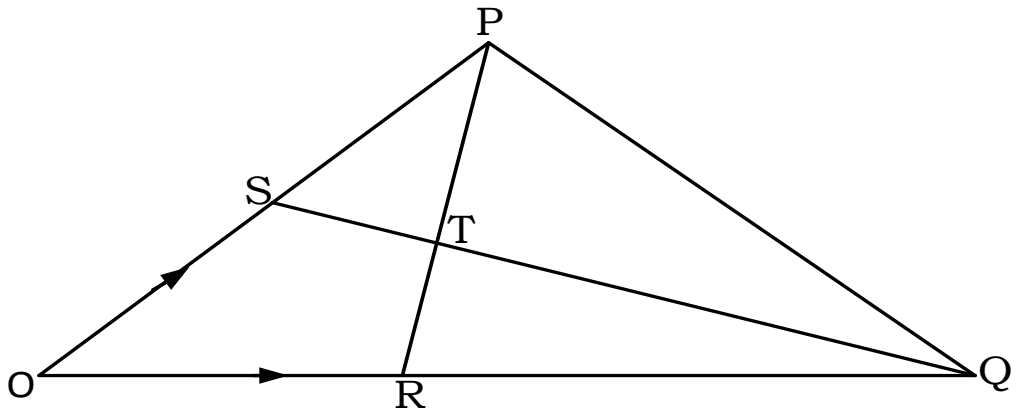
Express.

(i) Vectors $\overrightarrow{AM}$ and $\overrightarrow{OS}$ in terms of a , b and x

(ii) $\overrightarrow{OL}$ and $\overrightarrow{OS}$ in terms of $\boldsymbol{a}, \boldsymbol{b},$ and $\boldsymbol{y}$.

(iii) Hence find the values of constants x and y.

51. The figure below shows triangle OPQ in which, **OS** $= \frac{1}{3}\boldsymbol{OP}$ and $\boldsymbol{OR} = \frac{1}{3}\boldsymbol{OQ}$. T is a point on QS such that 4 **QT** = 3**QS.**



(a). Given, ***OP*** = ***p*** and **OQ** = ***q***, express the following vectors in terms of ***p*** and ***q*** ;

(i) **SR** (ii) **QS** (iii) **RT** (iv) **TP**

(b) Hence show that, R, T and P are collinear.

(T) TRANSFORMATION MATRICES

52. The coordinates of triangle PQR are P (2, -6), Q (5, -6) and R (2, -2). Using a scale of 1cm to represent 1 unit on both axes,

(a) Plot the triangle PQR.

(b) Using the line x + 2 = 0 as the mirror line, reflect the triangle PQR onto $P'Q'R'$. Write down the coordinates of $P'Q'R'$.

(c) Triangle $P'Q'R'$ is given a half turn about the origin onto $P''Q''$. Write down the coordinates of $P''Q''R''$

(d) Given that triangle $P''Q''R''$ is transformed onto $P''Q''R''$ by the transformational matrix M=,$\begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix}$, by calculation find the area of triangle $P''Q''R''$.

(U) KINEMATICS

53. John and Mike live 60km apart. At 8:00 am, John left his home cycling towards Mike's home at 25 kmh^{-1}. At 9:00 am, Mike left his home cycling towards John's home at 10 kmh^{-1}.

Calculate the;

(i) Time when the two men met

(ii) Distance from John's home to where the two men met.

54. Town A is **350km** away from Town B. At **6:00am** a Spacio car leaves town A for town B travelling at a steady speed of **50kmh^{-1}**. After travelling for an hour, the driver stopped on a highway restaurant for refreshments which took **30 minutes** before resuming his journey with the original steady speed. Two hours later from the time a Spacio car left town A bus travelling at a steady **non-stop** speed of **70kmh^{-1}** left town B for A.

(a) Using a scale of **2cm** to represent **1** hour and **2cm** to represent **50km**, draw on the same axes, graphs showing the journeys of the two vehicles.

(b) Using your graphs, determine the:

(i) Distance from town A where the two vehicles met.

(ii) Time when the two vehicles met.

(iii) Times when the vehicles arrive at their stations.

(iv) Difference in the times of arrivals of the vehicles at their respective stations.

(V) CONSTRUCTION

55. (a) Using a ruler and a pair of compasses only, construct a parallelogram PQRS in which angle PQR = 60^{o}, length QP=6cm and length QR=11cm.

(b) Drop a perpendicular from S to meet QR produced at T. Measure the length RT.

(c) Construct a circle passing through the vertices of triangle SRT. Measure its radius.

(d) Calculate the area of the three segments cut off by the triangle SRT i.e total area of the three segments.

(W) COORDINATE GEOMETRY

56. The points (-3,2), (4, k) and (0,2) lie on straight line. Find the value of k.

57. A straight-line M passing though points (-3,5) is perpendicular to another line N which passes through points (5,1) and (3, -2). Determine the equation of line M.

58. Given the line 9x-3y = 1. Find the;

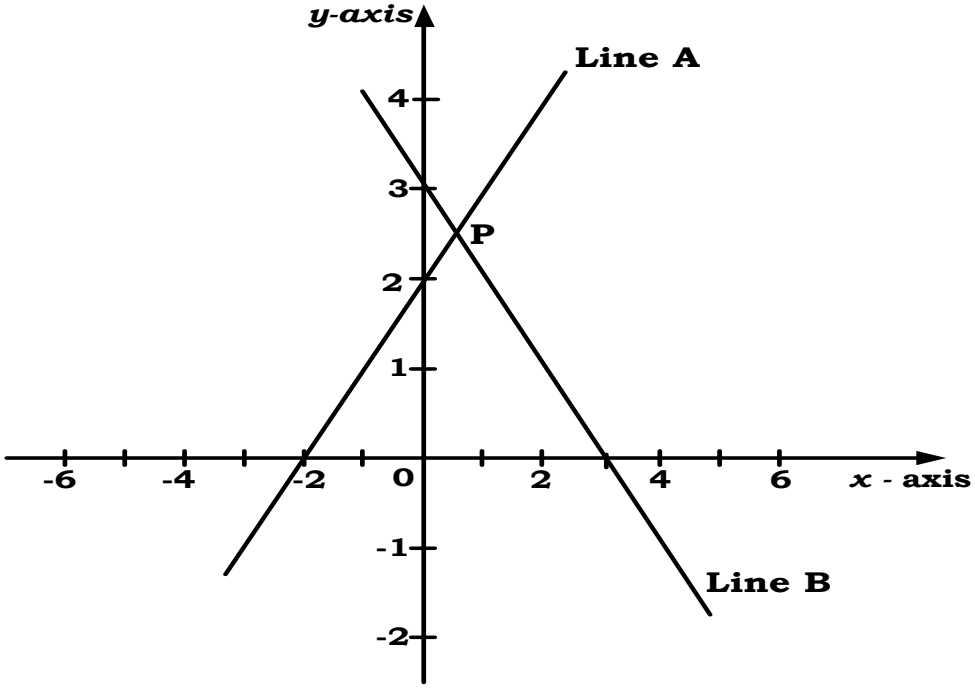
(i) gradient of a line.
(ii) x-intercept and y-intercept.

59. Find the equation of a line passing through the point (-3,2)

and is parallel to line 4y-3x-6 = 0.

60. The coordinates of points A and B are (-4,2) and (5, -1) respectively. If B is the midpoint of AC; determine the coordinates of C.

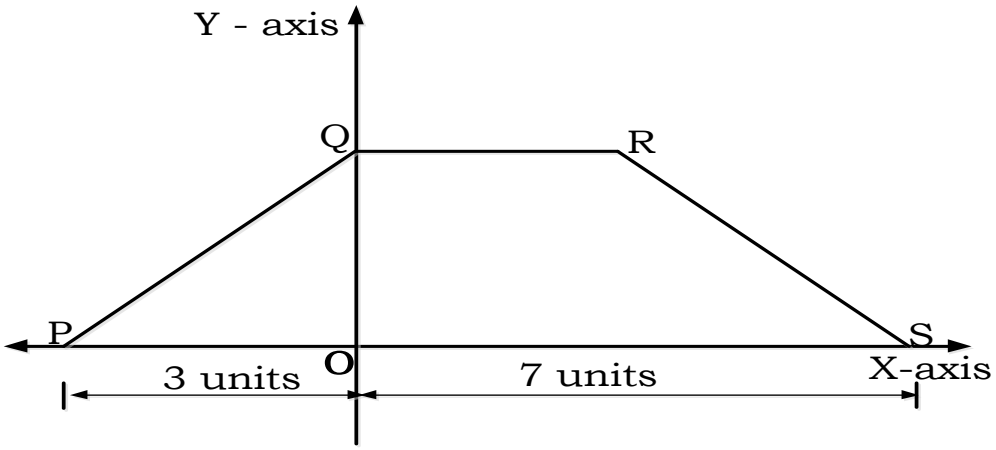
61. The sketch below is a graph of lines A and B intersecting at a point ,P.



(i) Find the coordinates of P, the point of intersection.

(ii) Find the area enclosed by lines A, B and line y =0.

62. The diagram below is an isosceles trapezium PQRS in which QR is parallel PS, RS = PQ and line QR has an equation of y = 6.



Find the;

(i) coordinates of points; P, Q, S and R.

(ii) |PS|

(iii) area of PQRS.

(X) RATIOS AND PROPORTIONS

63. Given that x varies directly as the cube of y and that x = 64 when y = 8. Find the value of;

(i) x when y = 12 (ii) y when x = 1

64. "*a*" varies directly as the square of "*b*" and inversely as "*c*". given that; *a* = 6, b = 4 and *c* = 2. Find *c, when;*

(i) *a* = 8 and3 *b* = 6

(ii) Find "*b*" when $a = \frac{3}{4}$ and *c* = 1.

65. (a) The table below shows values of "*p*" and the corresponding values of "*q*". Study the table and complete the missing details.

p	10	6	3	----
q	8.4	14	----	12

66. Sixteen men working at the rate of 9 hours a day can complete a piece of work in 14 days. How many men working at the rate of 7 hours a day would complete the same job in 12 days.

67. A map is drawn to a scale of 1:200,000. What area in km^2 on the ground is represented by a garden of area $5cm^2$ on a map.

(Y) VARIATIONS

68. In a certain NGO the cost of feeding its member per day is partly constant and partly proportional to the number of members in the organization (N). The cost of feeding 100 members is shs 500,000 per day. The cost of feeding 200 members is shs 800,000 per day.

    Calculate the;

    (i) cost of feeding 205 members per day.

    (ii) number of members needed to be fed on Ug sh. 449,000.

69. If x varies partly as y and partly as the square of y. if x = 16 when y = 2 and x = 33 when y = 3. Find the value of x when y = 5.

(Z) EQUATIONS

70. Solve for P in $\frac{P-3}{5} - \frac{P+2}{3} = \frac{P-6}{2} + 3$

71. A right-angled triangle has sides measuring 2x cm, (2x + 1) cm and (x + 1) cm. find the area of the triangle in m^2.

72. Mary's present age is $\frac{1}{4}$ of her mother's age. In eight years' time, she will be $\frac{1}{3}$ of her mothers age. Find their present age.

73. Solve the equation: $(x+2)^2 - (x-1) = 9$

THE END

THANK YOU

"SMATA"

Together for Mathematics

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